Documentation of the optimization routine

In the following we describe the configuration of the optimization solver which should help the reader to understand our code in more detail. The main challenge of the optimization was to make the numerical routine feasible. First, in order to increase convergence rate it is necessary to scale all variables to a similar magnitude and set bounds accordingly. Second, we supply gradients of our objective function and equilibrium constraints and form a jacobian matrix. Both adjustments result in a considerable speed gain and enable us to solve the optimization problem in appropriate time on a high-end workstation (one optimization takes $\sim$ 2-4 hours). Before we discuss the gradients, we first state the maximization problem of our routine.

Stating the maximization problem

The Matlab file *cons_eq.m* contains information on all equilibrium equation (CA_n)-(CG_n), whereas the file $func.m$ transmits the value of the objective function (OBJ_1) to the solver. Note that we specify the maximization problem in terms of regional transfer shares TS_n such that $TS_n\bar{T} = T_n$. The following description of the maximization problem applies to all three types of transfers. In the case of wage transfers TS_n refers to the regional share of wage transfers i.e. $TS_n\bar{T} = T_n^w$, while it refers to the regional shares of investments in production amenities and transport infrastructure for the other types, respectively.

$$\min_{\lambda_n, w_n, \pi_{nn}, TS_n, V_1, \bar{T}} \quad -V_1 \tag{OBJ_1}$$

subject to:

$$w_n - \alpha \frac{1}{\lambda_n}\left(\frac{w_n}{a_n}\right)^{1-\sigma} \sum_{i\in N}\left(\frac{d_{in}a_i}{d_{ii}w_i}\right)^{1-\sigma} \pi_{ii} y_i \lambda_i = 0, \qquad \forall \quad n \in N \tag{CA_n}$$

$$\pi_{nn} - \frac{\left(\frac{d_{nn}w_n}{a_n}\right)^{1-\sigma}}{\sum_{i\in N}\left(\frac{d_{ni}w_i}{a_i}\right)^{1-\sigma}} = 0, \qquad \forall \quad n \in N \tag{CB_n}$$

$$\lambda_n - \frac{B_n\left(\frac{a_n y_n}{d_{nn}w_n}\right)^{\alpha\epsilon}(\pi_{nn})^{\alpha\epsilon/(1-\sigma)}\left(\frac{H_n}{\lambda_n}\right)^{(1-\alpha)\epsilon}}{\sum_{i\in N} B_i\left(\frac{a_i y_i}{d_{ii}w_i}\right)^{\alpha\epsilon}(\pi_{ii})^{\alpha\epsilon/(1-\sigma)}\left(\frac{H_i}{\lambda_i}\right)^{(1-\alpha)\epsilon}} = 0, \qquad \forall \quad n \in N \tag{CC_n}$$

$$V_1 - \bar{L}^{-(1-\alpha)}\delta\gamma\,(B_1)^{\frac{1}{\epsilon}}\left(\frac{a_1 y_1}{d_{11}w_1}\right)^{\alpha}(\pi_{11})^{\alpha/(1-\sigma)}\left(\frac{H_1}{\lambda_1}\right)^{1-\alpha}\left(\frac{1}{\lambda_1}\right)^{\frac{1}{\epsilon}} = 0 \tag{CD_1}$$

$$\sum_{i\in N} \lambda_i - 1 = 0 \tag{CE_1}$$

$$\sum_{i\in N} TS_i - 1 = 0 \tag{CF_1}$$

$$\sum_{i\in N} w_i \lambda_i \bar{L} \tau_i - \bar{T} = 0 \tag{CG_1}$$

where

$$y_n = \frac{1}{\alpha + \iota_n - \alpha\iota_n}\left(w_n(1-\tau_n) + \left(\frac{TS_n\bar{T}}{\lambda_n\bar{L}}\right)\kappa^w + \chi\right), \forall \quad n \in N$$

$$a_n = \tilde{a}_n\left(\frac{TS_n\bar{T}}{\lambda_n\bar{L}} + 1\right)^{\kappa^a}\left(\lambda_n\bar{L}\right)^{\mu} \quad ,\forall \quad n \in N$$

$$d_{ni} = \Gamma\left(\frac{\theta-1}{\theta}\right)\left[\mathbf{I} - \tilde{\mathbf{D}}\right]_{ni}^{\frac{1}{\theta}},$$

$$TravelTime_{ri} = \gamma_{ri}^d - \kappa^d \cdot ln\left(TS_r\bar{T} + TS_i\bar{T} + 1\right). \tag{D.5}$$

In the counterfactuals of optimal distributions of:

- Wage subsidies $\kappa^a = \kappa^d = 0$ and $\kappa^w = 1$
- Investments in production amenities $\kappa^w = \kappa^d = 0$ and $\kappa^a > 0$
- Investments in transportation infrastructure $\kappa^w = \kappa^a = 0$ and $\kappa^d > 0$.

Supplying gradients to the solver

For our optimization problem we compute the Jacobian matrix for all variables $x_n \in \{\lambda_n, w_n, \pi_{nn}, TS_n, V_1, \bar{T}\}$, all our constraints $C \in \{CA_n - CG_1\}$ and our objective function $\{OBJ_1\}$. The matrix $J_{C,x}$ shows the jacobian matrix of constraint C with respect to variable variable x.

$$J_{C,x} = \begin{bmatrix} \frac{\partial C_1}{\partial x_1} & \dots & \frac{\partial C_n}{\partial x_1} & \dots & \frac{\partial C_N}{\partial x_1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial C_1}{\partial x_n} & \dots & \frac{\partial C_n}{\partial x_n} & \dots & \frac{\partial C_N}{\partial x_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial C_1}{\partial x_N} & \dots & \frac{\partial C_n}{\partial x_N} & \dots & \frac{\partial C_N}{\partial x_N} \end{bmatrix}.$$

The Matlab file *cons_gradients.m* forms the overall Jacobian of our equation system. All corresponding derivatives are described further below. First, it is instructive to clarify the link between the notation used below and our code. Partial derivatives with respect to the own regions attribute i.e. $\frac{\partial y_k}{\partial x_k}$ are reflected by the full matrix in the Matlab code, whereas derivatives with respect to other regions attributes i.e. $\frac{\partial y_n}{\partial x_k}$ are only positive for the main diagonal such that a matrix $SAME$ is multiplied in the code. All own-region variables i.e. x_n are transposed whereas all non-own-region variables x_k are not transposed. A superscript $(^*)$ denotes for each variable an equilibrium value. In order to isolate a particular transfer channels we set the partial derivatives that are relevant for the respective transfer type to the following expressions:

- For wage subsidies we activate (D.46)-(D.51) and (D.65)
- For investments in production amenities we activate (D.59)-(D.64) and (D.47), (D.52)
- For investments in transportation infrastructure we activate (D.53)-(D.58), (D.47), (D.52) and (D.65) where we compute the Jacobian of constraints J_{C,TS_n} and $J_{C,\bar{T}}$ using numerical automatic differentiation methods.

In the following, we state the Jacobian matrix for the equation system, which we supply to the solver

$$\frac{\partial CA_n}{\partial \lambda_k} = \frac{w_n^*}{\lambda_n}\frac{\partial \lambda_n}{\partial \lambda_k} + (1-\sigma)\frac{w_n^*}{a_n}\frac{\partial a_n}{\partial a_k} - \frac{\alpha}{\lambda_n}\left(\frac{w_n}{a_n}\right)^{1-\sigma}\left(\frac{d_{kn}a_k}{d_{kk}w_k}\right)^{1-\sigma}\pi_{kk}y_k\lambda_k\left(\frac{(1-\sigma)}{a_k}\frac{\partial a_k}{\partial \lambda_k} + \frac{1}{y_k}\frac{\partial y_k}{\partial \lambda_k} + \frac{1}{\lambda_k}\frac{\partial \lambda_k}{\partial \lambda_k}\right) \tag{D.6}$$

$$\frac{\partial CA_n}{\partial w_k} = \frac{\partial w_n}{\partial w_k} - (1-\sigma)\frac{w_n^*}{w_n}\frac{\partial w_n}{\partial w_k} - \frac{\alpha}{\lambda_n}\left(\frac{w_n}{a_n}\right)^{1-\sigma}\left(\frac{d_{kn}a_k}{d_{kk}w_k}\right)^{1-\sigma}\pi_{kk}y_k\lambda_k\left(-\frac{(1-\sigma)}{w_k}\frac{\partial w_k}{\partial w_k} + \frac{1}{y_k}\frac{\partial y_k}{\partial w_k}\right) \tag{D.7}$$

$$\frac{\partial CA_n}{\partial \pi_{kk}} = -\frac{\alpha}{\lambda_n}\left(\frac{w_n}{a_n}\right)^{1-\sigma}\left(\frac{d_{kn}a_k}{d_{kk}w_k}\right)^{1-\sigma}y_k\lambda_k\frac{\partial \pi_{kk}}{\partial \pi_{kk}} \tag{D.8}$$

$$\frac{\partial CA_n}{\partial TS_k} = (1-\sigma)\frac{w_n^*}{a_n}\frac{\partial a_n}{\partial TS_k} - \frac{\alpha}{\lambda_n}\left(\frac{w_n}{a_n}\right)^{1-\sigma}\left(\frac{d_{kn}a_k}{d_{kk}w_k}\right)^{1-\sigma}\pi_{kk}y_k\lambda_k \times\left(\frac{(1-\sigma)}{a_k}\frac{\partial a_k}{\partial TS_k} + \frac{1}{y_k}\frac{\partial y_k}{\partial TS_k}\right)$$

$$\frac{\partial CA_n}{\partial V_1} = 0 \tag{D.9}$$

$$\frac{\partial CA_n}{\partial \bar{T}} = (1-\sigma)\frac{w_n^*}{a_n}\lambda_n\frac{\partial a_n}{\partial \bar{T}} - \frac{\alpha}{\lambda_n}\left(\frac{w_n}{a_n}\right)^{1-\sigma}\sum_{k\in N}\left(\frac{d_{kn}a_k}{d_{kk}w_k}\right)^{1-\sigma}\pi_{kk}y_k\lambda_k \times\left(\frac{(1-\sigma)}{a_k}\frac{\partial a_k}{\partial \bar{T}} + \frac{1}{y_k}\frac{\partial y_k}{\partial \bar{T}}\right)$$

$$\frac{\partial CB_n}{\partial \lambda_k} = (1-\sigma)\frac{\pi^*_{nn}}{a_n}\frac{\partial a_n}{\partial \lambda_k} - (1-\sigma)\pi^*_{nn}\pi^*_{nk}\frac{1}{a_k}\frac{\partial a_k}{\partial \lambda_k} \tag{D.10}$$

$$\frac{\partial CB_n}{\partial w_k} = -(1-\sigma)\frac{\pi^*_{nn}}{w_n} + (1-\sigma)\pi^*_{nn}\pi^*_{nk}\frac{1}{w_k}\frac{\partial w_k}{\partial w_k} \tag{D.11}$$

$$\frac{\partial CB_n}{\partial \pi_{kk}} = \frac{\partial \pi_{nn}}{\partial \pi_{kk}} \tag{D.12}$$

$$\frac{\partial CB_n}{\partial TS_k} = (1-\sigma)\pi^*_{nn}\left(\frac{1}{a_n}\frac{\partial a_n}{\partial TS_k}\right) \tag{D.13}$$

$$- (1-\sigma)\pi^*_{nn}\pi^*_{nk}\left(\frac{1}{a_k}\frac{\partial a_k}{\partial TS_k}\right)$$

$$\frac{\partial CB_n}{\partial V_1} = 0 \tag{D.14}$$

$$\frac{\partial CB_n}{\partial \bar{T}} = (1-\sigma)\pi^*_{nn}\left(\frac{1}{a_n}\frac{\partial a_n}{\partial \bar{T}}\right) \tag{D.15}$$

$$- (1-\sigma)\pi^*_{nn}\sum_{k\in N}\pi_{nk}\left(\frac{1}{a_k}\frac{\partial a_k}{\partial \bar{T}}\right)$$

$$\frac{\partial CC_n}{\partial \lambda_k} = \frac{\partial \lambda_n}{\partial \lambda_k} + (1-\alpha)\epsilon\lambda_n^* \left(\frac{1}{\lambda_n}\frac{\partial \lambda_n}{\partial \lambda_k} - \frac{\lambda_k^*}{\lambda_k}\frac{\partial \lambda_k}{\partial \lambda_k} \right)$$
$$- \alpha\epsilon\lambda_n^* \left(\frac{1}{a_n}\frac{\partial a_n}{\partial \lambda_k} - \frac{\lambda_k^*}{a_k}\frac{\partial a_k}{\partial \lambda_k} + \frac{1}{y_n}\frac{\partial y_n}{\partial \lambda_k} - \frac{\lambda_k^*}{y_k}\frac{\partial y_k}{\partial \lambda_k} \right) \tag{D.16}$$

$$\frac{\partial CC_n}{\partial w_k} = \alpha\epsilon\lambda_n^* \left(\frac{1}{w_n}\frac{\partial w_n}{\partial w_k} - \frac{\lambda_k^*}{w_k} - \frac{1}{y_n}\frac{\partial y_n}{\partial w_k} + \frac{\lambda_k^*}{y_k}\frac{\partial y_k}{\partial w_k} \right) \tag{D.17}$$

$$\frac{\partial CC_n}{\partial \pi_{kk}} = -\frac{\alpha\epsilon}{1-\sigma}\lambda_n^* \left(\frac{1}{\pi_{nn}}\frac{\partial \pi_{nn}}{\partial \pi_{kk}} - \frac{\lambda_k^*}{\pi_{kk}}\frac{\partial \pi_{kk}}{\partial \pi_{kk}} \right) \tag{D.18}$$

$$\frac{\partial CC_n}{\partial TS_k} = -\alpha\epsilon\lambda_n^* \left(\frac{1}{a_n}\frac{\partial a_n}{\partial TS_k} - \frac{\lambda_k^*}{a_k}\frac{\partial a_k}{\partial TS_k} + \frac{1}{y_n}\frac{\partial y_n}{\partial TS_k} - \frac{\lambda_k^*}{y_k}\frac{\partial y_k}{\partial TS_k} \right) \tag{D.19}$$

$$\frac{\partial CC_n}{\partial V_1} = 0 \tag{D.20}$$

$$\frac{\partial CC_n}{\partial \bar{T}} = -\alpha\epsilon\lambda_n^* \left(\frac{1}{a_n}\frac{\partial a_n}{\partial \bar{T}} - \sum_{k\in N}\frac{\lambda_k^*}{a_k}\frac{\partial a_k}{\partial T} + \frac{1}{y_n}\frac{\partial y_n}{\partial \bar{T}} - \sum_{k\in N}\frac{\lambda_k^*}{y_k}\frac{\partial y_k}{\partial \bar{T}} \right) \tag{D.21}$$

$$\frac{\partial CD_1}{\partial \lambda_k} = \frac{(1-\alpha)(1-\sigma)\epsilon + (1-\sigma)}{(1-\sigma)\epsilon} \frac{V_1^*}{\lambda_1} \frac{\partial \lambda_1}{\partial \lambda_k} - \frac{\alpha V_1^*}{a_1} \frac{\partial a_1}{\partial \lambda_k} - \frac{\alpha V_1^*}{y_1} \frac{\partial y_1}{\partial \lambda_k} \tag{D.22}$$

$$\frac{\partial CD_1}{\partial w_k} = -\alpha V_1^* \left(\frac{1}{y_1} \frac{\partial y_1}{\partial w_k} - \frac{1}{w_1} \frac{\partial w_1}{\partial w_k} \right) \tag{D.23}$$

$$\frac{\partial CD_1}{\partial \pi_{kk}} = -\frac{\alpha}{1-\sigma} \frac{V_1^*}{\pi_{11}} \frac{\partial \pi_{11}}{\partial \pi_{kk}} \tag{D.24}$$

$$\frac{\partial CD_1}{\partial TS_k} = \alpha V_1^* \left(-\frac{1}{a_1} \frac{\partial a_1}{\partial TS_k} - \frac{1}{y_1} \frac{\partial y_1}{\partial TS_k} \right) \tag{D.25}$$

$$\frac{\partial CD_1}{\partial V_k} = 1 \tag{D.26}$$

$$\frac{\partial CD_1}{\partial \bar{T}} = \alpha V_1^* \left(-\frac{1}{a_1} \frac{\partial a_1}{\partial \bar{T}} - \frac{1}{y_1} \frac{\partial y_1}{\partial \bar{T}} \right) \tag{D.27}$$

$$\frac{\partial CE_1}{\partial \lambda_k} = 1 \tag{D.28}$$

$$\frac{\partial CE_1}{\partial w_k} = 0 \tag{D.29}$$

$$\frac{\partial CE_1}{\partial \pi_{kk}} = 0 \tag{D.30}$$

$$\frac{\partial CE1}{\partial TS_k} = 0 \tag{D.31}$$

$$\frac{\partial CE_1}{\partial V_1} = 0 \tag{D.32}$$

$$\frac{\partial CE_1}{\partial \bar{T}} = 0 \tag{D.33}$$

$$\frac{\partial CF_1}{\partial \lambda_k} = 0 \tag{D.34}$$

$$\frac{\partial CF_1}{\partial w_k} = 0 \tag{D.35}$$

$$\frac{\partial CF_1}{\partial \pi_{kk}} = 0 \tag{D.36}$$

$$\frac{\partial CF_1}{\partial TS_k} = 1 \tag{D.37}$$

$$\frac{\partial CF_1}{\partial V_1} = 0 \tag{D.38}$$

$$\frac{\partial CF_1}{\partial \bar{T}} = 0 \tag{D.39}$$

$$\frac{\partial CG_1}{\partial \lambda_k} = w_k \tau_k \bar{L} \tag{D.40}$$

$$\frac{\partial CG_1}{\partial w_k} = \tau_k \lambda_k \bar{L} \tag{D.41}$$

$$\frac{\partial CG_1}{\partial \pi_{kk}} = 0 \tag{D.42}$$

$$\frac{\partial CG_1}{\partial TS_k} = 0 \tag{D.43}$$

$$\frac{\partial CG_1}{\partial V_k} = 0 \tag{D.44}$$

$$\frac{\partial CG_1}{\partial \bar{T}} = -1 \tag{D.45}$$

In addition to the partial derivatives of the Jacobian, we express the partial derivatives of the variables capturing the direct effects of transfers.

$$\frac{\partial y_n}{\partial \lambda_k} = -\frac{1}{\alpha + \iota_n - \alpha\iota_n}\left(\frac{TS_n\bar{T}}{\lambda_n^2\bar{L}}\right)\frac{\partial \lambda_n}{\partial \lambda_k} \tag{D.46}$$

$$\frac{\partial y_n}{\partial w_k} = \frac{1}{\alpha + \iota_n - \alpha\iota_n}(1-\tau_n)\frac{\partial w_n}{\partial w_k} \tag{D.47}$$

$$\frac{\partial y_n}{\partial \pi_{kk}} = 0 \tag{D.48}$$

$$\frac{\partial y_n}{\partial TS_k} = \frac{1}{\alpha + \iota_n - \alpha\iota_n}\left(\frac{\bar{T}}{\lambda_n\bar{L}}\right)\frac{\partial TS_n}{\partial TS_k} \tag{D.49}$$

$$\frac{\partial y_n}{\partial V_1} = 0 \tag{D.50}$$

$$\frac{\partial y_n}{\partial \bar{T}} = \frac{1}{\alpha + \iota_n - \alpha\iota_n}\left(\frac{TS_n}{\lambda_n\bar{L}}\right) \tag{D.51}$$

If no wage subsidies are paid, then (D.51) becomes

$$\frac{\partial y_n}{\partial \bar{T}} = \frac{1}{\alpha + \iota_n - \alpha\iota_n}\left(\frac{1}{\bar{L}}\right) \tag{D.52}$$

$$\frac{\partial d_{ni}}{\partial \lambda_k} = 0 \tag{D.53}$$

$$\frac{\partial d_{ni}}{\partial w_k} = 0 \tag{D.54}$$

$$\frac{\partial d_{ni}}{\partial \pi_{kk}} = 0 \tag{D.55}$$

$$\frac{\partial d_{ni}}{\partial TS_k} = A.Diff. \tag{D.56}$$

$$\frac{\partial d_{ni}}{\partial V_1} = 0 \tag{D.57}$$

$$\frac{\partial d_{ni}}{\partial \bar{T}} = A.Diff. \tag{D.58}$$

$$\frac{\partial a_n}{\partial \lambda_k} = -\tilde{a}_n \kappa^a \left(\frac{TS_n \bar{T}}{\lambda_n \bar{L}} + 1\right)^{\kappa^a - 1} \left(\frac{TS_n \bar{T}}{\lambda_n^2 \bar{L}}\right) \left(\lambda_n \bar{L}\right)^{\mu} \frac{\partial \lambda_n}{\partial \lambda_k} + a_n^* \frac{\mu}{\lambda_n} \frac{\partial \lambda_n}{\partial \lambda_k} \tag{D.59}$$

$$\frac{\partial a_n}{\partial w_k} = 0 \tag{D.60}$$

$$\frac{\partial a_n}{\partial \pi_{kk}} = 0 \tag{D.61}$$

$$\frac{\partial a_n}{\partial TS_k} = \tilde{a}_n \kappa^a \left(\frac{TS_n \bar{T}}{\lambda_n \bar{L}} + 1\right)^{\kappa^a - 1} \left(\lambda_n \bar{L}\right)^{\mu} \left(\frac{\bar{T}}{\lambda_n \bar{L}}\right) \tag{D.62}$$

$$\frac{\partial a_n}{\partial V_1} = 0 \tag{D.63}$$

$$\frac{\partial a_n}{\partial \bar{T}} = \tilde{a}_n \kappa^a \left(\frac{TS_n \bar{T}}{\lambda_n \bar{L}} + 1\right)^{\kappa^a - 1} \left(\lambda_n \bar{L}\right)^{\mu} \left(\frac{TS_n}{\lambda_n \bar{L}}\right) \tag{D.64}$$

If no investments in production amenities are paid then (D.59) becomes

$$\frac{\partial a_n}{\partial \lambda_k} = a_n^* \frac{\mu}{\lambda_n} \frac{\partial \lambda_n}{\partial \lambda_k} \tag{D.65}$$

Finally, the derivative of the objective function is as follows

$$\frac{\partial OBJ_1}{\partial \lambda_k} = 0 \tag{D.66}$$

$$\frac{\partial OBJ_1}{\partial w_k} = 0 \tag{D.67}$$

$$\frac{\partial OBJ_1}{\partial \pi_{kk}} = 0 \tag{D.68}$$

$$\frac{\partial OBJ_1}{\partial TS_k} = 0 \tag{D.69}$$

$$\frac{\partial OBJ_1}{\partial V_1} = -1 \tag{D.70}$$

$$\frac{\partial OBJ_1}{\partial \bar{T}} = 0 \quad . \tag{D.71}$$